

Name:

Solutions / Answers

1. Given the function $f(x) = xe^x$, use the first derivative and the second derivative to determine:

- a. The interval(s) on which the function is increasing,

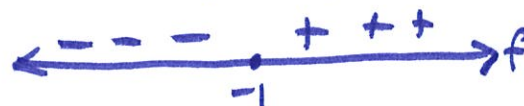
$$f'(x) = e^x + xe^x = e^x(1+x)$$

$$e^x \neq 0 \quad 1+x=0 \quad x=-1 \text{ c.p.}$$

$f(x)$ is increasing on $[-1, \infty)$

since $f'(x) > 0$

$$0 = e^x(1+x)$$



- b. The interval(s) on which the function is decreasing,

referring to the sign analysis above,

$f(x)$ is decreasing on $(-\infty, -1]$ since $f'(x) < 0$

- c. The x value of any point of inflection,

$$f''(x) = e^x + e^x + xe^x = 2e^x + xe^x = e^x(2+x)$$

$$f''(x) = e^x(2+x) \quad e^x \neq 0 \quad 2+x=0 \quad x=-2$$

$f(x)$ has a p. of I. at $x=-2$ since $f''(-2)=0$

and f'' changes sign at $x=-2$.



- d. The interval(s) on which the graph is concave up,

Referring to the sign analysis for f'' above,

$f(x)$ is concave up on $(-2, \infty)$ since $f''(x) > 0$

- e. The interval(s) on which the graph is concave down.

Referring to the sign analysis for f'' above,

$f(x)$ is concave down on $(-\infty, -2)$ since $f''(x) < 0$

- f. Use the second derivative test to identify any minimum or maximum values.


$$x=-1 \text{ is a c.p.} \quad f(-1) = -e^{-1} = -\frac{1}{e}$$

$f(x)$ has an abs. min of $y = -\frac{1}{e}$ at $x=-1$ since

$f''(-1) > 0$ in which case the graph of $f(x)$ is concave up at $x=-1$.

2. Given the function $f(x) = 8x - x^2$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing,

$$f'(x) = 8 - 2x \quad 0 = 8 - 2x \quad x = 4 \text{ c.p.}$$


$f(x)$ is increasing on $(-\infty, 4]$ since $f'(x) > 0$ on $(-\infty, 4)$

b. The interval(s) on which the function is decreasing,

Referring to a. above, $f(x)$ is decreasing on $[4, \infty)$ since $f'(x) < 0$ on $(4, \infty)$

c. The x value of any point of inflection,

$f''(x) = -2$ $f''(x) < 0$ for all real x
 $f''(x)$ does not change sign; therefore, $f(x)$ is always concave down; therefore, $f(x)$ does not have a p. of I.

d. The interval(s) on which the graph is concave up,

$f''(x) < 0$ for all real x
 $f(x)$ is never concave up since $f''(x) < 0$

e. The interval(s) on which the graph is concave down,


Referring to d. above, $f(x)$ is concave down on $(-\infty, \infty)$ since $f''(x) < 0$

f. Use the second derivative test to identify any minimum or maximum values.

$$f(4) = 16$$

$x = 4$ is a c.p. $f''(4) < 0$ $f(x)$ is concave down
 $f(x)$ has an abs. max of $y = 16$ at $x = 4$
 since $f'(4) = 0$ and $f''(4) < 0$.

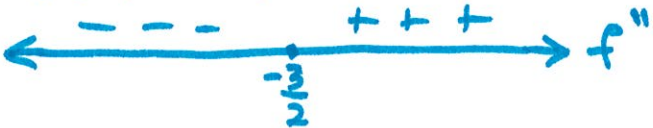
3. Given the function $f(x) = x^3 + \frac{9}{2}x^2 - 12x - 2$, use the first derivative and the second derivative to determine:

a. The interval(s) on which the function is increasing, $f'(x) = 3x^2 + 9x - 12$
 $0 = 3(x^2 + 3x - 4) = 3(x+4)(x-1)$ $x = -4$ c.p. $x = 1$ c.p.

 $f(x)$ is increasing on $(-\infty, -4] \cup [1, \infty)$ since $f'(x) > 0$

- b. The interval(s) on which the function is decreasing,

Referring to a. above,
 $f(x)$ is decreasing on $[-4, 1]$ since $f'(x) < 0$

- c. The x value of any point of inflection,

$f''(x) = 6x + 9$ $0 = 6x + 9$ $x = -\frac{3}{2}$ c.p.

 $f(x)$ has a p. of I at $x = -\frac{3}{2}$ since $f''(-\frac{3}{2}) = 0$ and f'' changes sign at $x = -\frac{3}{2}$

- d. The interval(s) on which the graph is concave up,

Referring to c. above,
 $f(x)$ is concave up on $(-\frac{3}{2}, \infty)$ since $f''(x) > 0$ on $(-\frac{3}{2}, \infty)$

- e. The interval(s) on which the graph is concave down,

Referring to c. above,
 $f(x)$ is concave down on $(-\infty, -\frac{3}{2})$ since $f''(x) < 0$ on $(-\infty, -\frac{3}{2})$

- f. Use the second derivative test to identify any minimum or maximum values.

$$f(-4) = -64 + 72 + 48 - 2 = 54$$

$$f(1) = 1 + \frac{9}{2} - 12 - 2 = -8.5$$

$f(x)$ has a local min of $y = -8.5$ at $x = 1$ since $f''(1) > 0$
 $f(x)$ has a local max of $y = 54$ at $x = -4$ since $f''(-4) < 0$

4. Given the function $f(x) = x^4 - 6x^2 + 12$, use the first derivative and the second derivative to determine:

- a. The interval(s) on which the function is increasing, $f'(x) = 4x^3 - 12x$
 $0 = 4x(x^2 - 3)$ $x = 0$ c.p. $x = \sqrt{3}$ c.p. $x = -\sqrt{3}$ c.p.

$f(x)$ is increasing on $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$ since $f'(x) \geq 0$

- b. The interval(s) on which the function is decreasing,

Referring to a. above,
 $f(x)$ is decreasing on $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$ since $f'(x) \leq 0$

- c. The x value of any point of inflection,

$f''(x) = 12x^2 - 12 = 12(x-1)(x+1)$ $0 = 12(x-1)(x+1)$
 $x = 1$ $x = -1$

 $f(x)$ has p. of I at $x = -1$ and at $x = 1$ since $f''(-1) = 0$ and $f''(1) = 0$ and $f''(x)$ changes sign at $x = -1$ and $x = 1$.

- d. The interval(s) on which the graph is concave up,

Referring to c. above,
 $f(x)$ is concave up on $(-\infty, -1) \cup (1, \infty)$ since $f''(x) > 0$

- e. The interval(s) on which the graph is concave down,

Referring to c. above,
 $f(x)$ is concave down on $(-1, 1)$ since $f''(x) < 0$

- f. Use the second derivative test to identify any minimum or maximum values.

$f(-\sqrt{3}) = 3$ $f(0) = 12$ $f(\sqrt{3}) = 3$
 $f(x)$ has an abs. min. of $y = 3$ at $x = -\sqrt{3}$ and $x = \sqrt{3}$ since $f'(-\sqrt{3}) = 0$ and $f''(-\sqrt{3}) > 0$ and since $f'(\sqrt{3}) = 0$ and $f''(\sqrt{3}) > 0$.
 $f(x)$ has a local max of $y = 12$ at $x = 0$ since $f'(0) = 0$ and $f''(0) < 0$